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Estimasi Satu Parameter Distribusi Weibull Pada Model Data Survival Tersensor

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Abstrak

Penelitian ini mengelaborasi untuk menemukan estimasi parameter, fungsi hazard dan fungsi survival dari distribusi Weibull satu parameter dengan menggunakan metode Maksimum Like lihood untuk data tersensor. Hasilnya menunjukkan bahwa estimasi dari parameter adalah is**Error! Reference source not found.**. Selanjutnya juga diperoleh estimator dari fungsi survival dan fungsi yaiut **Error! Reference source not found.**and **Error! Reference source not found.**.

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Kata Kunci: distribusi Weibull, MLE

Abstract

This paper elaborates a research which finding the estimators of the parameter, hazard function and survival function of the one-parameterweibull distribution by using maximum likelihood estimation method on cencored data. The result shows that estimator of parameter is**Error! Reference source not found.**. Furthermore, we got the estimators of survival function and hazard function are**Error! Reference source not found.**and **Error! Reference source not found.**, respectively.

Keywords :

PENDAHULUAN

One of special kind of variable random is survival model. Let **Error! Reference source not found.** is defined to be the time of failure of the entity known to exist at time **Error! Reference source not found.**, and is therefore frequently called the failure time random variable. Now if **Error! Reference source not found.** is the time to failure, then the probability of still functioning at time **Error! Reference source not found.** is the same as the probability that the failure is later (mathematically greater) than the value of **Error! Reference source not found.**. Formally,

$$S(t) = Pr(T > t)$$

By the nature of **Error! Reference source not found.** is clear that **Error! Reference source not found.**, that **Error! Reference source not found.**, and that **Error! Reference source not found.** is a non-increasing function. We will asssume**Error! Reference source not found.**. The Cumulative Distribution Function of **Error! Reference source not found.**is **Error! Reference source not found.**. That is **Error! Reference source not found.**
it should be clear that **Error! Reference source not found.** and that **Error! Reference source not found.** and **Error! Reference source not found.**.

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For the special case of a continuous random variable, the probability density function, **Error! Reference source not found.**, is defined as the derivative of **Error! Reference source not found.**. Thus,

$$f(t) = \frac{d}{dt}F(t) = -\frac{d}{dt}S(t) \ , \qquad t \geq 0$$

and then we defined the hazard rate function as **Error! Reference source not found.**

METODE PENELITIAN

The Weibull distribution is a known distribution which can be used in survival studies. It is a very popular one-parameter distribution which is often used in survival model study. Let **Error! Reference source not found.** is defined to be the time of failure of the entity known to exist at time **Error! Reference source not found.**, and is therefore frequently called the failure time random variable. Now, if **Error! Reference source not found.** is the time to failure, then the probability of still functioning at time **Error! Reference source not found.** is the same as the probability that the failure is later (mathematically greater) than the value of **Error! Reference source not found.**. The survival density function (SDF), probability density function (PDF) and hazard rate function (HRF) are defined as below,

$$S(t) = e^{-\left(\frac{kt^{n+1}}{n+1}\right)}$$

$$f(t) = kt^n e^{-\left(\frac{kt^{n+1}}{n+1}\right)}$$

$$\lambda(t) = kt^n$$

Cencoring is a way to handle an uncomplete data which is caused some events like death. loss or out from observation. Variables **Error! Reference source not found.**represent **Error! Reference source not found.** individual lifetimes. A time **Error! Reference source not found.** is the lifetime or a cencoring time. The variable **Error! Reference source not found.** if **Error! Reference source not found.** and **Error! Reference source not found.** if **Error! Reference source not found.** is called the cencoring or status indicator for **Error! Reference source not found.**. Value **Error! Reference source not found.** is obtained from **Error! Reference source not found.** where **Error! Reference source not found.** is the duration of their remission measured from time of entry to study and **Error! Reference source not found.** is the time between their date of entry and the end of study. The likelihood function of cencored data for observation **Error! Reference source not found.** can be calculated is defined as,

$$L(t_i; k, \delta) = \prod_{i=1}^{m} [f(t_i; k)]^{\delta_i} [S(t_i; k)]^{1-\delta_i}$$

HASIL DAN PEMBAHASAN

The likelihood function of weibull distribution for observation **Error! Reference source not found.** can be calculated by

$$L(t_i; \lambda, \delta) = \prod_{i=1}^{n} \left[k(t_i)^n e^{-\left(\frac{k(t_i)^{n+1}}{n+1}\right)} \right]^{\delta_i} \left[e^{-\left(\frac{k(t_i)^{n+1}}{n+1}\right)} \right]^{1-\delta_i}$$

$$= \{[k(t_1)^n]^{\delta_1}[k(t_2)^n]^{\delta_2} \dots [k(t_m)^n]^{\delta_m}\}e^{-\left(\frac{k(\sum_{i=1}^m t_i)^{n+1}}{n+1}\right)}$$

$$= k^{\sum_{i=1}^m \delta_m}\{(t_1)^{n\delta_1}(t_2)^{n\delta_2} \dots (t_m)^{n\delta_m}\}e^{-\left(\frac{k(\sum_{i=1}^m t_i)^{n+1}}{n+1}\right)}$$

$$= \left[\prod_{i=1}^m (t_i)^{n\delta_i}\right] k^{\sum_{i=1}^m \delta_m} e^{-\left(\frac{k(\sum_{i=1}^m t_i)^{n+1}}{n+1}\right)}$$

then we find a natural logaritme of likelihood function above,

$$l = \ln L(t_i; k, \delta) = \left(\left[\prod_{i=1}^m (t_i)^{n\delta_i}\right] k^{\sum_{i=1}^m \delta_m} e^{-\left(\frac{k(\sum_{i=1}^m t_i)^{n+1}}{n+1}\right)}\right)$$

$$= n\sum_{i=1}^m \delta_i \ln t_i + \sum_{i=1}^m \delta_m \ln k - \frac{k(\sum_{i=1}^m t_i)^{n+1}}{n+1}$$

by deriving **Error! Reference source not found.** to parameter **Error! Reference source not found.**. weobtain,

$$\frac{dl}{dk} = 0$$

$$\frac{d}{dk}\left[n\sum_{i=1}^m \delta_i \ln t_i + \sum_{i=1}^m \delta_i \ln k - \frac{k(\sum_{i=1}^m t_i)^{n+1}}{n+1}\right] = 0$$

$$\frac{\sum_{i=1}^m \delta_i}{k} - \frac{(\sum_{i=1}^m t_i)^{n+1}}{n+1} = 0$$

$$\hat{k} = \frac{(n+1)\sum_{i=1}^m \delta_i}{\left(\sum_{i=1}^m t_i\right)^{n+1}}$$

KESIMPULAN DAN SARAN

We get **Error! Reference source not found.** is a Maximum Likelihood Estimation of **Error! Reference source not found.**. Later,composing **Error! Reference source not found.** into both of survival model and hazard function are

$$\hat{S}_{ML}(t_i; \hat{k}) = e^{-\hat{k}t_i} = e^{-\left(\frac{(n+1)(\sum_{i=1}^m \delta_i)t_i^{n+1}}{(n+1)\left(\sum_{i=1}^m t_i\right)^{n+1}}\right)}$$

$$= e^{-\left(\frac{\sum_{i=1}^m \delta_i}{(\sum_{i=1}^m t_i)^{n+1}}\right)t_i^{n+1}}$$

$$\hat{h}_{ML}(t_i; \hat{\lambda}) = \hat{k}(t_i)^n = \frac{(n+1)\sum_{i=1}^m \delta_i}{\left(\sum_{i=1}^m t_i\right)^{n+1}}(t_i)^n$$

Error! Reference source not found. and **Error! Reference source not found.** are Maximum Likelihood Estimation of survival model and hazard function.

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DAFTAR PUSTAKA

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